Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9999967514 5

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

[Turn over

1 It is given that θ is an acute angle in degrees such that $\sin\theta = \frac{2}{3}$.

Find the exact value of $\sin(\theta + 60°)$. [3]

2 A curve has equation $y = 3\tan\frac{1}{2}x\cos 2x$.

Find the gradient of the curve at the point for which $x = \frac{1}{3}\pi$. [5]

3 **(a)** Find $\int_4^{10} \frac{4}{2x-5}\,\mathrm{d}x$, giving your answer in the form $\ln a$, where a is an integer. [4]

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(b) Find the exact value of $\int_4^{10} \mathrm{e}^{2x-5}\,\mathrm{d}x$. [2]

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4 **(a)** Sketch, on the same diagram, the graphs of $y = |3x - 5|$ and $y = 2x + 7$. [2]

(b) Solve the equation $|3x - 5| = 2x + 7$. [3]

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(c) Hence solve the equation $\left|3^{y+1} - 5\right| = 2 \times 3^{y} + 7$, giving your answer correct to 3 significant figures. [2]

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5 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 6x^3 + ax^2 + bx - 20,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and that the remainder is -11 when $\mathrm{p}(x)$ is divided by $(x + 1)$.

(a) Find the values of a and b. [5]

(b) Hence factorise $p(x)$, and determine the exact roots of the equation $p(3x) = 0$. [4]

6 **(a)** Show that $\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) \equiv 4 + 6\cos\theta - 4\cos^2\theta$. [3]

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(b) Solve the equation

$$\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) + 3 = 0$$

for $-\pi < \theta < 0$. [3]

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(c) Find $\int \operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta)\,\mathrm{d}\theta$. [3]

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7 The curve with equation $e^{2x} - 18x + y^3 + y = 11$ has a stationary point at (p, q).

(a) Find the exact value of p. [4]

(b) Show that $q = \sqrt[3]{2 + 18 \ln 3 - q}$. [2]

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(c) Show by calculation that the value of q lies between 2.5 and 3.0. [2]

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(d) Use an iterative formula, based on the equation in **(b)**, to find the value of q correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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